XI. *On the Parallax of the Fixed Stars.* *By Mr.* Herſchel, *F. R. S.*; *communicated by Sir* Joſeph Banks, *Bart. P. R. S.*

Read December 6, 1781.

TO find the diſtance of the fixed ſtars has been a problem which many eminent aſtronomers have attempted to ſolve; but about which, after all, we remain in a great meaſure ſtill in the dark. Various methods have been purſued without ſucceſs, and the reſult of the fineſt obſervations has hardly given us more than a diſtant approximation, from which we may conclude, that the neareſt of the fixed ſtars cannot be leſs than forty thouſand diameters of the whole annual orbit of the earth diſtant from us. Trigonometry, by whoſe powerful aſſiſtance the mathematician has boldly aſcended into the planetary regions, and meaſured the diameters and orbits of the heavenly bodies, for want of a proper baſe, can here be but of little ſervice; for the whole diameter of the annual orbit of the earth is a mere point when compared to the immenſe diſtance of the ſtars. Now, as it is not in our power to enlarge this baſe, we can only endeavour to improve the inſtruments by which we meaſure its parallax.

There are two things requiſite for meaſuring extremely ſmall angles with accuracy. Firſt, that the inſtrument we uſe for this purpoſe, be it quadrant, ſector, or micrometer, ſhould be divided and executed with ſufficient exactneſs; and, ſecondly, that the teleſcope, by which the obſervations are to be made,

made, ſhould have an adequate power and diſtinctneſs. Upon the firſt head, the great improvements of mathematical inſtrument-makers have hardly left us any thing to deſire: we can now meaſure ſeconds with almoſt as much facility and truth as former obſervers could meaſure minutes; nor do I think it impoſſible to go ſtill further, and divide inſtruments that would ſhew thirds with ſufficient accuracy. It is in the latter, or optical part, we find the greateſt difficulty. To ſee a ſingle ſecond of a degree with preciſion requires a teleſcope of very great perfection; therefore, ſuppoſing the mechanical part of an apparatus well executed, it will ſtill be neceſſary to try how far the power of our teleſcope will enable us to aſcertain with confidence the diviſion or number of ſeconds it points out. If upon trial we find that our inſtrument will give us the ſame meaſure within the ſecond, every time the experiment is repeated, we may pronounce it capable of meaſuring ſeconds; if otherwiſe, it will remain to be examined, whether the fault lies in the mechanical or optical part.

Let us now ſuppoſe that the parallax of the fixed ſtars does not amount to a ſingle ſecond, yet ſtill the caſe is by no means deſperate; and though the difficulty of meaſuring ſeconds will ſoon ſuggeſt to us what extraordinary powers and diſtinctneſs of the teleſcope, and accuracy of the micrometer, are required to meaſure thirds; this ought by no means to diſcourage us in the attempt. Could we meaſure angles, much ſmaller than ſeconds, might we not hope to find the parallax of ſome of the fixed ſtars at leaſt to amount to ſeveral thirds? On the other hand, if it ſhould appear, indeed, that even with ſuch improved methods of meaſurement we could not reach the remote ſituation of ſuch almoſt infinitely diſtant ſuns, we might ſtill derive a valuable approximation towards truth from

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ſuch repeated obſervations, even though they ſhould not be attended with all the ſucceſs we expected from them. On this aſſurance, I endeavoured to take ſuch a method for attempting the inveſtigation of the parallax of the ſtars as to avail myſelf of the improvements I had already made, and was ſtill in hopes of making, in my teleſcopes.

The next thing that was neceſſary to conſider in this undertaking was, the manner of putting it into execution. The method pointed out by GALILEO, and firſt attempted by HOOK, FLAMSTEAD, MOLINEUX, and BRADLEY, of taking diſtances of ſtars from the zenith that paſs very near it, though it failed with regard to parallax, has been productive of the moſt noble diſcoveries of another nature. At the ſame time it has given us a much juſter idea of the immenſe diſtance of the ſtars, and furniſhed us with an approximation to the knowledge of their parallax that is much nearer the truth than we ever had before. Dr. BRADLEY, in a letter to Dr. HALLEY on the ſubject of a new diſcovered motion of the fixed ſtars, ſays, " I believe I may venture to ſay, that in either of the " two ſtars laſt mentioned (γ Draconis and η Urſæ majoris) it " (the annual parallax) does not amount to $2''$. I am of opi- " nion, that if it were $1''$ I ſhould have perceived it in the " great number of obſervations that I made, eſpecially upon " γ Draconis; which agreeing with the hypotheſis (without " allowing any thing for parallax) nearly as well when the " ſun was in conjunction with, as in oppoſition to, this ſtar, " it ſeems very probable, that the parallax of it is not ſo great " as one ſingle ſecond." Phil. Tranſ. n. 406. p. 637. Dec. 1728. As I do not know that any thing more deciſive has been done upon the ſubject, it will not be amiſs to ſee how far this method of finding the parallax has really been ſucceſsful.

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The inſtrument that was uſed upon this occaſion was the ſame as the preſent zenith ſectors, which can hardly be allowed ſufficient to ſhew an angle of one or even two ſeconds with accuracy; yet, on account of the great number of obſervations, and above all the great ſagacity of the obſerver, we will admit that if the parallax had amounted to two ſeconds he would have perceived it. The ſtar on which theſe obſervations were made is marked of the third magnitude in the catalogue of PTOLEMY; in TYCHO BRAHE's of the third; in the Prince of HESSE's of the third; in HEVELIUS's between the third and ſecond; in FLAMSTEAD's of the ſecond; and now appears as a very bright ſtar of the third, or ſmall ſtar of the ſecond magnitude; therefore its parallax is probably conſiderably leſs than that of a ſtar of the firſt magnitude. Several authors who have touched upon this ſubject ſeem to have overlooked this diſtinction; and from Dr. BRADLEY's account of the parallax of γ Draconis, have concluded the parallax of the ſtars in general not to exceed $1''$; but this appears to me by no means to follow from the doctor's obſervations. It is rather evident that, for aught we know to the contrary, the ſtars of the firſt magnitude may ſtill have a parallax of ſeveral ſeconds; and I believe this to be as accurate a reſult as that method is capable of giving, at leaſt in latitudes where there is not a ſtar of the firſt magnitude that paſſes directly through the zenith *.

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* DE LA LANDE, in his excellent book of Aſtronomy, ſays, that the parallax of the fixed ſtars has been proved to be abſolutely inſenſible (Aſt. liv. XVI. § 2782.). He reports the obſervations of TYCHO BRAHE, PICARD, HOOK, and FLAMSTEAD, and concludes (§ 2778.) from the diſcovery of the aberration by Dr. BRADLEY (which it ſeems he alſo allows to be the moſt deciſive upon the ſubject) that now the queſtion about parallax is reſolved. In giving us the opinion which the

In general, the method of zenith diftances labours under the following confiderable difficulties. In the firft place, all thefe diftances, though they fhould not exceed a few degrees, are liable to refractions; and I hope to be pardoned when I fay that the real quantities of thefe refractions, and their differences,

the doctor had of the refult of his own obfervations with regard to the annual parallax, DE LA LANDE only mentions " M. BRADLEY penfe que fi elle (la " parallaxe) eût été feulement de 1″ il l'auroit apperçue dans le grand " nombre d'obfervations qu'il avoit faites, furtout de γ du Dragon." But if we alfo take in thofe lines upon which Dr. BRADLEY feems to lay the greateft ftrefs, viz. " I believe I may venture to fay, that in either of the " two ftars laft mentioned it does not amount to two feconds;" and if we allow for the magnitude of the ftars upon which the obfervations were made, I think I have fairly ftated the full amount of all the actual proofs we have of the fmallnefs of the annual parallax. Now, fince it has efcaped the fineft obfervations of BRADLEY, it is not likely that it fhould come up to the full quantity to which it might amount without being perceived; and therefore the doctor might think it highly probable, " that it is not fo great as one fingle fecond;" and his opinion, as well as DE LA LANDE's, who believes it to be abfolutely infenfible, are perfectly confiftent with all the obfervations that have hitherto been made; though the *actual proofs*, which are the fubject of our prefent inquiry, do not extend fo far. Againft the parallax of Sirius DE LA LANDE (§ 2781.) mentions " forty-" five meridian altitudes taken by Dr. BEVIS [a], with the eight-feet mural qua-" drant of the Royal Obfervatory at Greenwich, none of which differed 3 or 4″ " from the mean altitude." Now, if they differed 3 or 4″ from the mean, we may fuppofe they differed 6 or 8″ from each other; and that obfervations, fubject to fo many caufes of error as I fhall prefently enumerate, and which differed fo much from each other, cannot give the leaft evidence either for or againft a parallax, will need no proof. Refraction alone, which is liable to fuch changes at the meridian altitude of Sirius, notwithftanding the moft careful obfervations of the barometer and thermometer fhould be made to afcertain its quantity, would, with me, remain an unanfwerable argument againft the validity of fuch obfervations in a fubject of this critical nicety.

[a] Thefe obfervations were not made by Dr. BEVIS, but extracted from the regifters of the Royal Obfervatory at my defire, and calculated by myfelf, and fent in a letter by Dr. BEVIS to Paris.

NEVIL MASKELYNE.

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are very far from being perfectly known. Secondly, the change of position of the earth's axis arising from nutation, precession of the equinoxes, and other causes, is so far from being completely settled, that it would not be very easy to say what it exactly is at any given time. In the third place, the aberration of light, though best known of all, may also be liable to some small errors, since the observations from which it was deduced laboured under all the foregoing difficulties. I do not mean to say, that our theories of all these causes of error are defective; on the contrary, I grant that we are for most astronomical purposes sufficiently furnished with excellent tables to correct our observations from the above mentioned errors. But when we are upon so delicate a point as the parallax of the stars; when we are investigating angles that may, perhaps, not amount to a single second, we must endeavour to keep clear of every possibility of being involved in uncertainties; even the hundredth part of a second becomes a quantity to be taken into consideration.

I shall now deliver the method I have taken, and shew that it is free from every error to which the former is liable, and is still capable of every improvement the telescope and mechanism of micrometers can furnish.

Let OE (fig. 1.) be two opposite points of the annual orbit, taken in the same plane with two stars a, b, of unequal magnitudes. Let the angle aOb be observed when the earth is at O: and let the angle aEb be also observed when the earth is at E. From the difference of these angles, if any should be found, we may calculate the parallax of the stars, according to a theory that will be delivered hereafter. These two stars, for reasons that will soon appear, ought to be as near each other as possible,

poſſible, and alſo to differ as much in magnitude as we can find them.

GALILEO, I believe, was the firſt who ſuggeſted this method; but in the manner he mentions it in his third dialogue of the *Syſtema Coſmicum*, it would be expoſed to all the difficulties we have enumerated, and would wiſh to avoid; for he does not obſerve, that the two ſtars ſhould be ſo near each other as thereby to preclude the influence of every cauſe of error.

This method has alſo been mentioned by other authors; and we find that Dr. LONG obſerved the double ſtar which is the firſt of Aries in PTOLEMY's catalogue; that in the head of Caſtor; the middle one in the ſword of Orion; and that in the breaſt of Virgo, with teleſcopes of fourteen and ſeventeen feet, and "was perſuaded they would be found always to appear "the ſame." But when the theory of parallax will be explained, it will be ſeen that every one of theſe ſtars are totally improper for the purpoſe; for the ſtars of γ Arietis are near $10''$ diſtant from each other, and moreover equal in magnitude. In α Geminorum the ſtars, though near enough, do not ſufficiently differ in magnitude to ſhew any parallax. The ſtars in the Nebula of Orion, on account of their extreme ſmallneſs or diſtance, are ſtill more improper than any; and thoſe of γ Virginis are equal in magnitude.

I do not find that any thing elſe has been done upon the ſubject. GALILEO juſtly remarks, that ſuch obſervations ought to be made with the beſt teleſcopes, and upon this occaſion mentions the power of his own, which enlarged the diſk of the ſun a thouſand times, from which we find it magnified about thirty-two times; but we can hardly think his nor even Dr. LONG's, whoſe power might probably be ſixty or ſeventy, ſufficient for the purpoſe. What would GALILEO ſay, if he were told that our

our preſent opticians make inſtruments that enlarge the diſk of the ſun above forty thouſand times? What would even CASSINI ſay, if he were to view the firſt ſtar of Aries, which appeared to him as ſplit in two, through a teleſcope that will ſhew η Coronæ borealis and *h* Draconis to be double ſtars?

But to proceed, I ſhall now prove that this method, if ſtars properly ſituated (ſuch as I have found) are taken, is free from all the errors occaſioned by refraction, nutation, preceſſion of the equinoxes, changes of the obliquity of the ecliptic, and aberration of light; and that the annual parallax, if it even ſhould not exceed the tenth part of a ſecond, may ſtill become viſible, and be aſcertained at leaſt to a much greater degree of approximation than it ever has been done.

It will alſo appear, from the great number of obſervations I have already made upon ſeveral double ſtars, eſpecially ϵ Bootis, that we can now with much greater certainty affirm the annual parallax to be exceedingly ſmall indeed; and that there is a great probability of ſucceeding ſtill farther in this laborious but delightful reſearch, ſo as to be able at laſt to ſay, not only how much the annual parallax *is not*, but how much it really *is*.

Let there be two ſtars at a diſtance from each other, not exceeding five ſeconds; ſuppoſe them to be obſerved at an altitude of 20°; and let them be ſo ſituated with reſpect to each other, that one of them may be 20°, and the other 20° and 5′ high: then the whole effect of mean refraction at that altitude, by Dr. MASKELYNE's excellent tables, will be 2′ 35″,5 for 20°, and 2′ 35″,4888 for 20° 5″. The difference is 0″,0111. Now, in the firſt place, we have nothing to do with the refraction itſelf, ſince the real altitude of the ſtars is not in queſtion. In the next place, we alſo have no concern with the difference of refrac-

tion between the two ſtars, though no more than the ,0111th part of a ſecond, becauſe the real diſtance between the two ſtars is not required. It follows then, that theſe obſervations can only be affected by the difference of the difference; that is, by an alteration in the quantity of refraction occaſioned by the change of heat and cold, or weight of the atmoſphere, and pointed out to us by the riſe and fall of the barometer and thermometer. Let us then ſee what this difference of the difference may amount to. Suppoſe a change of 22° of FAHRENHEIT's thermometer, that is, from the freezing point to the moderate air of a ſummer's night, and a difference of an inch in the height of the barometer; theſe two cauſes both conſpiring, which does not often happen, may occaſion an alteration of ,00096th part of a ſecond in five, at an altitude of 20°; but this being leſs than the thouſandth part of a ſecond may ſafely be rejected as a quantity altogether inſenſible.

Since it may not be always convenient to view thoſe ſtars at the altitude of 20°, it remains to ſee what effect different altitudes may have: let us then make the moſt unfavourable ſuppoſition, that they may one time be ſeen in a horizontal poſition, having before been ſeen vertical. In this caſe, as the whole difference of refraction in a difference of 5″ of altitude is no more than ,0111, provided they are obſerved not lower than 20°, and the whole difference of the difference of refraction is only ,0009; the ſum ,012, when both conſpire, not exceeding much the hundredth part of a ſecond, may ſtill be rejected as inſenſible. Let us alſo examine how near the horizon it may be ſafe to obſerve ſuch ſtars. At 10°, for inſtance, the refraction is 5′ 14″,6; the difference for 5″ is ,0388; the joint effect of the changes in the barometer and thermometer is ,0034; the ſum of the whole together amounts to ,0422, which

is leſs than half the tenth of a ſecond: now this may either be taken into conſideration, or ſuch low obſervations may be avoided, as being by no means neceſſary, and but ill ſuiting the high powers a teleſcope proper for this purpoſe ought to bear.

The change of poſition of the earth's axis I look upon as an unſurmountable obſtacle to taking the parallax of ſtars by the method of zenith diſtances: for though refraction is much reduced in the zenith, this change is there no leſs ſenſible than in other parts of the heavens; but as this will always affect our two ſtars exactly alike, we are entirely freed from this embarraſſment.

The aberration of light can have no influence of the leaſt conſideration upon our two ſtars, as a mere inſpection of the tables will ſhew. In a whole degree, its effects, when greateſt, amount but to four-tenths of a ſecond, and conſequently in $5''$ to no more than ,0005, or the two thouſandth part of a ſecond.

Obſervations of the relative diſtance of the two ſtars that make up a double ſtar, being thus cleared of every impediment, are capable of being continually improved by every degree of perfection the teleſcope may acquire: we can chuſe ſtars that may be viewed ſufficiently high to be clear of the vapours that ſwim near the horizon, and conſequently employ the greateſt powers our inſtruments are capable of. From experience I can alſo affirm, that the ſtars will bear a much higher degree of magnifying than other celeſtial objects. Too much has hitherto been taken for granted in optics: every natural philoſopher is ready enough to allow the neceſſity of making experiments, and tracing out the ſteps of nature; why this method ſhould not be more purſued in the art of ſeeing

does not appear. Theories are only to be uſed when proper data are aſſigned; but the data are carefully to be re-examined, when new improvements may widely alter the reſult of former experiments. Thus, we are told, that we gain nothing by magnifying *too much.* I grant it; but ſhall never believe I magnify too much till by experience I find, that I can ſee better with a lower power. Nor is even that ſufficient: a lower power may ſhew more of the object; it may ſhew it brighter, nay even diſtincter, and therefore upon the whole better; and yet the greater power may, in a particular caſe, be preferable: for if the object is ſo ſmall as not to be at all viſible with the lower power, and I can, by magnifying more, obtain a view of it, though neither ſo bright nor diſtinct as I could wiſh, is it not evident, that here this power is preferable to the former?

The naturaliſt does not think himſelf obliged to account for all the phænomena he may obſerve; the aſtronomer and optician may claim the ſame privilege. When we increaſe the power we leſſen the light in the inverſe ratio of the ſquare of the power; and teleſcopes will, in general, diſcover more ſmall ſtars the more light they collect; yet with a power of 227 I cannot ſee the ſmall ſtar near the ſtar following *o* Aquilæ, when, by the ſame teleſcope, it appears very plainly with the power of 460: now, in the latter caſe, the power being more than double, the light is leſs than the fourth part of the former. In ſuch particular caſes I generally ſuſpect my own eyes, and have recourſe to thoſe of my friends. I had the pleaſure of ſhewing this ſtar to Dr. WATSON junior, who ſoon diſcovered the ſmall ſtar, which accompanies the other, with the power of 460; but ſaw nothing of it with 227, though the place where to look for it had been pointed out to him by the higher power. The experiment has been too often

repeated

repeated to be doubtful, and has alſo been confirmed by others of nearly the ſame nature: for inſtance, the ſmalleſt of the two that accompany the ſtar near *k* Aquilæ, the ſmall ſtar near μ Herculis, and the ſmall ſtar near α Lyræ, are inviſible with my power of 227, and viſible with the ſame aperture when the power is 460. Alſo the ſmall ſtars near FLAMSTEAD's 24th of Aquila, the ſmalleſt of two near σ Coronæ, the ſmall ſtar near the ſtar ſouth of ε Aquilæ, the ſmall ſtar near the ſecond *o* Perſei, the ſmall ſtar near the ſtar which accompanies FLAMSTEAD's 10th ſub pede et ſcapula dextra Tauri, the ſmall ſtar, near β Delphini, and the ſmall ſtar near the pole ſtar, are all much brighter and ſtronger, and therefore much ſooner ſeen with 460 than with 227.

Great power may alſo, in particular circumſtances, be favourable, even with an exceſs of aberration. When two ſtars are ſo cloſe together as to make the ſcale for meaſuring the diſtance of their centers too ſmall, if, by magnifying much, we can enlarge that diſtance, we may gain a conſiderable advantage, provided the centers or apparent bodies of the ſtars remain diſtinct enough for the purpoſe of theſe meaſures. The appearance of α Lyræ in my Newtonian reflector with a power of 460 is repreſented in fig. 2.; with 2010 in fig. 3.; with 3168 in fig. 4.; and with 6450 in fig. 5. Now in all theſe figures we ſee, that the centers are ſtill diſtinct enough to meaſure their diſtances with ſufficient truth; or if any little error ſhould be introduced by the magnitude of the central point, it will be more than ſufficiently balanced by the largeneſs of the ſcale. In this manner, with a power of 3168, I have obtained a ſcale of no leſs than ten inches ſix tenths for the diſtance of the centers of the two ſtars of α Geminorum; and as we

know theſe centers to be but a few ſeconds diſtant, it is plain how great an advantage we gain by ſuch an enlarged ſcale.

Theſe experiments have but very lately pointed out to me a method of making a new micrometer, upon a conſtruction entirely different from any that are now in uſe, which I have been ſucceſsful enough to put in practice, and by which I have already begun to determine the diſtance of the centers of ſome of the moſt remarkable double ſtars to a very great degree of accuracy *.

The powers that may be uſed upon various double ſtars are different, according to their relative magnitudes: ε Bootis, for inſtance, will not bear the ſame power as α Geminorum, nor would it be difficult to aſſign a reaſon for it; but as I here ſhall merely confine myſelf to facts, it will be ſufficient in general to mention, that two ſtars, which are equal, or nearly ſo, will bear a very high power: with α Geminorum I have gone as far as 3168; but with the former only to 2010. The difficulty of uſing high powers is exceedingly great; for the field of view takes in leſs than the diameter of the hair or wire in the finder, and the effect of the earth's diurnal motion is ſo great, that it requires a great deal of practice to find the object, and manage the inſtrument. It appears to me very probable, that the diurnal motion of the earth will be the greateſt obſtacle to our progreſs in magnifying, except we can introduce a proper mechaniſm to carry our teleſcopes in a contrary motion.

Notwithſtanding opticians have proved that two eye-glaſſes will give a more correct image than one, I have always (from experience) perſiſted in refuſing the aſſiſtance of a ſecond glaſs, which is ſure to introduce errors greater than thoſe we would correct. Let us reſign the double eye-glaſs to thoſe who view objects

* For a deſcription of this micrometer ſee a ſubſequent paper.

merely

merely for entertainment, and muſt have an exorbitant field of view. To a philoſopher this is an unpardonable indulgence. I have tried both the ſingle and double eye-glaſs of equal powers, and always found that the ſingle eye-glaſs had much the ſuperiority in point of light and diſtinctneſs. With the double eye-glaſs I could not ſee the *belts on Saturn*, which I very plainly ſaw with the ſingle one. I would, however, except all thoſe caſes where a large field is abſolutely neceſſary, and where power joined to diſtinctneſs is not the ſole object of our view.

The application of the different powers of a teleſcope in general is of ſome conſequence; and in anſwer to thoſe who may think I have ſtrained or over-charged mine, I muſt obſerve, that a ſingle glance at the ſubſequent *b* Draconis, η Coronæ, and the ſtar near μ Bootis, with a power of 460, ſhewed them to me as double ſtars; when, in two former reviews of the heavens, I had twice ſet them down in my journal as ſingle ſtars, where I uſed only the power of 222 and 227, and in all probability ſhould never have found them double, had I not looked with a higher power.

We are to remember, that it is much eaſier to ſee an object when it is pointed out to us than when it falls in our way unexpectedly, eſpecially if of ſuch a nature as to require ſome attention to be ſeen at all; but to ſay no more of other advantages of high powers, it is evident, that in the reſearch of the parallax of the fixed ſtars they are abſolutely neceſſary. If we would diſtinctly perceive and meaſure or eſtimate extremely ſmall quantities, ſuch as a tenth of a ſecond, it appears, that when we uſe a power of 460, this tenth of a ſecond will be no more in appearance than 46″, and even with a power of 1500 will be but 2′ 30″, which is a quantity not much more than

than ſufficient to judge well of objects and diſtinguiſh them from each other, ſuch as a circle from a ſquare, triangle, or polygon *.

It has been obſerved, that objects grow indiſtinct when the principal optic pencil at the eye becomes leſs than the 40th or 50th part of an inch in diameter. In the experiments that have been made upon this ſubject it appears to me, that the indiſtinctneſs which is aſcribed to the ſmallneſs of the optical pencil may be owing to very different cauſes: at leaſt it will be eaſy to bring contrary experiments of extremely ſmall pencils, not at all affected by this inconvenience; for inſtance, it is well known, that microſcopes, conſiſting of a ſingle lens or globule, are remarkable for diſtinctneſs. We alſo know, that they have been made ſo ſmall as to magnify above 10,000 times †. From this we may infer that their apertures, and conſequently the diameters of the optic pencil at the eye could not exceed the 2500th part of an inch. I am therefore inclined to believe, that we muſt look for diſtinctneſs in the perfection of the object-ſpeculum or object-glaſs of a teleſcope; and if we can make the firſt image in the focus of a ſpeculum almoſt as perfect as the real object, what ſhould hinder our magnifying but the want of light? Now, if the object has light ſufficient, as the ſtars moſt undoubtedly have, I ſee no reaſon why we ſhould limit the powers of our inſtruments by any theory. Is it not beſt to have recourſe to experiments to find how

* By a ſet of experiments, made in the year 1774, I found, that I could diſcover or perceive a bright object, ſuch as white paper, againſt the ſky-light, when it ſubtended an angle of 35″; but could only diſtinguiſh it to be a circle, and no other figure, when it appeared under an angle of 2′ 24″.

† See Padre DELLA TORRE's Method, &c. Scelta di Opuſculi.

how far our endeavours to render the firſt image perfect have been ſucceſsful.

As ſoon as I was fully ſatisfied that in the inveſtigation of parallax the method of double ſtars would have many advantages above any other, it became neceſſary to look out for proper ſtars. This introduced a new ſeries of obſervations. I reſolved to examine every ſtar in the heavens with the utmoſt attention and a very high power, that I might collect ſuch materials for this reſearch as would enable me to fix my obſervations upon thoſe that would beſt anſwer my end. The ſubject has already proved ſo extenſive, and ſtill promiſes ſo rich a harveſt to thoſe who are inclined to be diligent in the purſuit, that I cannot help inviting every lover of aſtronomy to join with me in obſervations that muſt inevitably lead to new diſcoveries. I took ſome pains to find out what double ſtars had been recorded by aſtronomers; but my ſituation permitted me not to conſult extenſive libraries, nor indeed was it very material: for as I intended to view the heavens myſelf, Nature, that great volume, appeared to me to contain the beſt catalogue upon this occaſion. However, I remembered that the ſtar in the head of Caſtor, that in the breaſt of the Virgin, and the firſt ſtar in Aries, had been mentioned by CASSINI as double ſtars. I alſo found the Nebula in Orion was marked in HUGEN's *Syſtema Saturnium* as containing ſeven ſtars, three of which (now known to be four) are very near together. With this ſmall ſtock I begun, and in the courſe of a few years obſervations have collected the ſtars contained in my catalogue. I find, with great pleaſure, that a very excellent obſerver, whom I have the honour to call my friend*, has alſo, though un-

* Phil. Tranſ. for the year 1781, part II. double ſtars diſcovered in 1779, at Frampton-houſe, Glamorganſhire, by NAT. PIGOTT, Eſq. F. R. S. &c.

 known

known to me, met with three of thoſe ſtars that will be found in my catalogue: and upon this occaſion I alſo beg leave to obſerve, that the Aſtronomer Royal, when I was at Greenwich laſt May, with his uſual politeneſs, ſhewed me, among other objects, α Herculis as a double ſtar, which he had diſcovered ſome years ago. The rev. Mr. HORNSBY alſo, when I had the pleaſure of ſeeing him at Oxford, in a converſation on the ſubject of the ſtars of the firſt magnitude that have a proper motion, mentioned π Bootis as a double ſtar. It is a little hard upon young aſtronomers to be obliged to diſcover *over-again* what has already been diſcovered; however, the pleaſure that attended the view when I firſt ſaw theſe ſtars has made ſome amends for not knowing they had been ſeen before me.

If I ſhould mention in my liſt of obſervations a few that may be found difficult to be verified by other teleſcopes, I muſt beg the indulgence of the obſervers. I hope it will ſufficiently appear, that I have guarded againſt optical deluſions; and every aſtronomer, I make no doubt, will find, by thoſe obſervations that fall within the compaſs of his inſtruments, and attention to circumſtances neceſſary to the right management of them, that I have had all along truth and reality in view, as the ſole object of my endeavours; and therefore he will be inclined to give ſome credit to what he does not immediately perceive, when he finds himſelf ſucceſsful where he takes the proper precautions ſo neceſſary in delicate obſervations, even with the beſt inſtruments.

I have been in ſome doubt in what manner to communicate theſe obſervations. My firſt view was to have methodized them properly; but I find them ſo extenſive that there is but little probability that one perſon ſhould be able to bring them to a concluſion, for which reaſon I have now reſolved to give them

 unfiniſhed

unfiniſhed as they are, that every perſon who is inclined to engage in this purſuit may become a fellow-labourer.

In ſettling the diſtances of double ſtars I have occaſionally uſed two different ways. Thoſe that are extremely near each other may be eſtimated by the eye, in meaſures of their own apparent diameters. For this purpoſe their diſtance ſhould not much exceed two diameters of the largeſt, as the eye cannot ſo well make a good eſtimation when the interval between them is greater. This method has often the preference to that of the micrometer: for inſtance, when the diameter of a ſmall ſtar, perhaps not equal to half a ſecond, is double the vacancy between the two ſtars. Here a micrometer ought to meaſure tenths of ſeconds at leaſt, otherwiſe we could not, with any degree of confidence, rely on its meaſures; nay, even then, if the ſtars are ſituated in the ſame parallel of declination and near the equator, their quick motion acroſs the micrometer makes it extremely difficult to meaſure them, and in that caſe an eſtimation by the eye is preferable to any other meaſure; but this requires not a little practice, precaution, and time, and yet with proper care it will be found that this method is capable of great exactneſs. Let two ſmall circles be drawn either equal or unequal, at a diſtance not exceeding twice the diameter of the largeſt; let theſe be ſhewn to ſeveral perſons in the ſame light and point of view. Then, if every one of them will ſeparately and carefully write down his eſtimation of the interval between them, in the proportion of either of their diameters, it will be found upon a compariſon that there will ſeldom be ſo much as a quarter of a diameter difference between all the eſtimations. If this agreement takes place with ſo many different eyes, much more may we expect it in the

eſtimations of the ſame eye when accuſtomed to this kind of judgement.

I have divided the double ſtars into ſeveral different claſſes. In the firſt I have placed all thoſe which require indeed a very ſuperior teleſcope, the utmoſt clearneſs of air, and every other favourable circumſtance to be ſeen at all, or well enough to judge of them. They ſeemed to me on that account to deſerve a ſeparate place, that an obſerver might not condemn his inſtrument or his eye if he ſhould not be ſucceſsful in diſtinguiſhing them.

As theſe are ſome of the fineſt, moſt minute, and moſt delicate objects of viſion I ever beheld, I ſhall be happy to hear that my obſervations have been verified by other perſons, which I make no doubt the curious in aſtronomy will ſoon undertake. I ſhould obſerve, that ſince it will require no common ſtretch of power and diſtinctneſs to ſee theſe double ſtars, it will therefore not be amiſs to go gradually through a few preparatory ſteps of viſion, ſuch as the following: when η Coronæ borealis (one of the moſt minute double ſtars) is propoſed to be viewed, let the teleſcope be ſome time before directed to α Geminorum, or if not in view to either of the following ſtars, ζ Aquarii, μ Draconis, ρ Herculis, α Piſcium, or the curious double-double ſtar ε Lyræ. Theſe ſhould be kept in view for a conſiderable time, that the eye may acquire the habit of ſeeing ſuch objects well and diſtinctly. The obſerver may next proceed to ξ Urſæ majoris, and the beautiful treble ſtar in Monoceros's right fore-foot; after theſe to ι Bootis, which is a fine miniature of α Geminorum, to the ſtar preceding α Orionis, and to η Orionis. By this time both the eye and the teleſcope will be prepared for a ſtill finer picture, which is η Coronæ borealis. It will be in vain to attempt this latter if all the former, at leaſt

i Bootis, cannot be diſtinctly perceived to be fairly ſeparated becauſe it is almoſt as fine a miniature of *i* Bootis as that is of *α* Geminorum. If the obſerver has been ſucceſsful in all theſe, he may then, at the ſame time, try *b* Draconis, though I queſtion whether any power leſs than 4 or 500 will ſhew it to be double; but the former I have all ſeen very well with 227.

To try the ſtars of unequal magnitudes it will be expedient to take them in ſome ſuch order as the following: *α* Herculis, *ω* Aurigæ, *δ* Geminorum, *k* Cygni, *ε* Perſei, and *b* Draconis; from theſe the obſerver may proceed to a moſt beautiful object, *ε* Bootis, which I have cloſely attended theſe two years as very proper for the inveſtigation of the parallax of the fixed ſtars.

It appears, from what has been ſaid, that theſe double ſtars are a moſt excellent way of trying a teleſcope; and as the foregoing remarks have ſuggeſted the method of ſeeing how far the power and diſtinctneſs of our inſtruments will reach, I ſhall add the way of finding how much light we have. The obſerver may begin with the pole-ſtar and *α* Lyræ; then go to the ſtar ſouth of *ε* Aquilæ, the treble ſtar near *k* Aquilæ, and laſt of all to the ſtar following *o* Aquilæ. Now, if his teleſcope has not a great deal of good diſtinct light, he will not be able to ſee ſome of the ſmall ſtars that accompany them.

In the ſecond claſs of double ſtars I have put all thoſe that are proper for eſtimations by the eye or very delicate meaſures of the micrometer. To compare the diſtances with the apparent diameters the power of the teleſcope ſhould not be much leſs than 200, as they will otherwiſe be too cloſe for the purpoſe. The inſtrument ought, moreover, to be as much as poſſible free from rays that ſurround a ſtar in common teleſcopes, and ſhould give the apparent diameters of a double ſtar perfectly round and well-defined, with a deep black diviſion

between

between them, as in fig. 6. which repreſents α Geminorum as I have often ſeen it with a power of 460. It will be neceſſary here to take notice, that the eſtimations made with one teleſcope cannot be applied to thoſe made with another: nor can the eſtimations made with different powers, though with the ſame teleſcope, be applied to each other. Whatever may be the cauſe of the apparent diameters of the ſtars, they are certainly not of equal magnitude with the ſame powers in different teleſcopes, nor of proportional magnitude with different powers in the ſame teleſcope. In my inſtruments I have ever found leſs diameter in proportion the higher I was able to go in power, and never have I found ſo ſmall a proportional diameter as when I magnified 6450 times*; therefore if we would wiſh to compare any ſuch obſervations together, with a view to ſee whether a change in the diſtance has taken place, it ſhould be done with the very ſame teleſcope and power, even with the very ſame eye-glaſs or glaſſes; for others, though of equal power and goodneſs, would moſt probably give different proportional diameters of the ſtars.

In the third claſs I have placed all thoſe double ſtars that are more than five but leſs than 15″ aſunder; and for that reaſon, if they ſhould be uſed for obſervations on the parallax of the fixed ſtars, they ought not to be looked upon as quite free from the effects of refraction, &c. In the ſame manner that the ſtars in the firſt and ſecond claſſes will ſerve to try the goodneſs of the moſt capital inſtruments, theſe will afford objects for teleſcopes of inferior power, ſuch as magnify from 40 to 100 times. The obſerver may take them in this or the like order: ζ Urſæ majoris, γ Delphini, γ Arietis, π Bootis, γ Virginis,

* See the meaſures of the diameter of α Lyræ. Catalogue of double ſtars, 5th claſs.

ginis, ι Caſſiopeæ, μ Cygni. And if he can ſee all theſe, he may paſs over into the ſecond claſs, and direct his inſtrument to ſome of thoſe that were pointed out as objects for the very beſt teleſcopes, where, I ſuppoſe, he will ſoon find the want of ſuperior power.

The fourth, fifth, and ſixth claſſes contain double ſtars that are from 15 to 30″, from 30″ to 1′, and from 1′ to 2′ or more aſunder. Though theſe will hardly be of any ſervice for the purpoſe of parallax, I thought it not amiſs to give an account of ſuch as I have obſerved; they may, perhaps, anſwer another very important end, which alſo requires a great deal of accuracy, though not quite ſo much as the inveſtigation of the parallax of the fixed ſtars. I will juſt mention it, though foreign to my preſent purpoſe. Several ſtars of the firſt magnitude have already been obſerved, and others ſuſpected, to have a proper motion of their own: hence we may ſurmiſe, that our ſun, with all its planets and comets, may alſo have a motion towards ſome particular part of the heavens, on account of a greater quantity of matter collected in a number of ſtars and their ſurrounding planets there ſituated, which may perhaps occaſion a gravitation of our whole ſolar ſyſtem towards it. If this ſurmiſe ſhould have any foundation, it will ſhew itſelf in a ſeries of ſome years; as from that motion will ariſe another kind of hitherto unknown parallax *, the inveſtigation of which may account for ſome part of the motions already obſerved in ſome of the principal ſtars; and for the purpoſe of determining the direction and quantity of ſuch a motion, accurate obſervations of the diſtance of ſtars that are near enough to be meaſured with a micrometer, and a very high power of

* See the note in the rev. Mr. MITCHELL's paper on the Parallax of the Fixed Stars, Phil. Tranſ. vol. LVII. p. 252.

teleſcopes,

telefcopes may be of confiderable ufe, as they will undoubtedly give us the relative places of thofe ftars to a much greater degree of accuracy than they can be had by tranfit inftruments or fectors, and thereby much fooner enable us to difcover any apparent change in their fituation occafioned by this new kind of fyftematical parallax, if I may be allowed to ufe that expreffion, for fignifying the change arifing from the motion of the whole folar fyftem.

I fhall now endeavour to deliver a theory of the annual parallax of double ftars, with the method of computing from thence what is generally called the parallax of the fixed ftars, or of fingle ftars of the firft magnitude, fuch as are neareft to us. It may be obferved, that the principles upon which I have founded the following theory are of fuch a nature, that they cannot be ftrictly demonftrated, in confequence of which they are only propofed as poftulata, which have fo great a probability in their favour, that they will hardly be objected to by thofe who are in the leaft acquainted with the doctrine of chances.

GENERAL POSTULATA.

1. Let the ftars be fuppofed, one with another, to be about the fize of the fun *.

2. Let the difference of their apparent magnitudes be owing to their different diftances, fo that a ftar of the fecond, third, or

* See Mr. MICHELL's Inquiry into the probable Parallax and Magnitude of the Fixed Stars, Phil. Tranf. vol. LVII. p. 234. 236. 237. 240. and Dr. HALLEY on the Number, Order, and Light, of the Fixed Stars, Phil. Tranf. vol. XXXI.

or fourth magnitude is two, three, or four times as far off as one of the firſt *.

In fig. 7. let OE be the whole diameter of the earth's annual orbit; and let a, b, c, be three ſtars ſituated in the ecliptic, in ſuch a manner that they may be ſeen all in one line Oabc, when the earth is at O. Let the line Oabc be perpendicular to OE, and draw PE parallel to cO. Then, if Oa, ab, bc, are equal to each other, a will be a ſtar of the firſt magnitude, b of the ſecond, and c of the third. Let us now ſuppoſe the angle OaE, or parallax of the whole orbit of the earth, to be $1''$ of a degree: then we have $PEa = OaE = 1''$.: and, becauſe very ſmall angles, having the ſame ſubtenſe OE, may be taken to be in the inverſe ratio of the lines Oa, Ob, Oc, &c. we ſhall have $ObE = \frac{1}{2}''$, $OcE = \frac{1}{3}''$, &c. †. Now, when the earth is removed

* The apparent magnitude is here taken in a ſtricter ſenſe than is generally uſed; and by it is rather meant the order into which the ſtars *ought to be* diſtinguiſhed than that into which they *are* commonly divided: for as the order of the magnitudes is here to denote the different relative diſtances, we are to examine carefully the degree of light each ſtar is accurately found to have: and conſidering then that light diminiſhes in the inverſe ratio of the ſquares of the diſtances, we ought to claſs the ſtars accordingly. An allowance ought alſo perhaps to be made for ſome loſs that may happen to the light of very remote ſtars in its paſſage through immenſe tracts of ſpace, moſt probably not quite deſtitute of ſome very ſubtle medium. This conjecture is ſuggeſted to us by the colour of the very ſmall teleſcopic ſtars, for I have generally found them red, or inclining to red; which ſeems to indicate, that the more feeble and refrangible rays of the other colours are either ſtopped by the way, or at leaſt diverted from their courſe by accidental deflections.

† This proves what I have before remarked on the parallax of γ Draconis; for that ſtar, (admitting it to be a ſtar of between the ſecond and third magnitude, which ought to be aſcertained by experiments, as mentioned in the note above) by the poſtulata, will have its place aſſigned ſomewhere between b and c, and therefore its parallax will be between $\frac{1}{2}$ and $\frac{1}{3}$ of the parallax of a ſtar of the firſt magnitude. And if Dr. BRADLEY thought that he ſhould have perceived a

paralx

removed to E, we ſhall have $PEb = EbO = \frac{1}{2}''$, and $PEa - PEb = aEb = \frac{1}{2}''$; that is, the ſtars a, b, will appear to be $\frac{1}{2}''$ diſtant. We alſo have $PEc = EcO = \frac{1}{3}''$, and $PEa - PEc = aEc = \frac{2}{3}''$; that is, the ſtars a, c, will appear to be $\frac{2}{3}''$ diſtant, when the earth is at E. Now, ſince we have $bEP = \frac{1}{2}''$, and $cEP = \frac{1}{3}''$, therefore $bEP - cEP = bEc = \frac{1}{2}'' - \frac{1}{3}'' = \frac{1}{6}''$; that is, the ſtars b, c, will appear to be only $\frac{1}{6}''$ removed from each other, when the earth is at E.

From what has been ſaid, we may gather the following general expreſſion, to denote the parallax that will become viſible in the change of diſtance between the two ſtars, by the removal of the earth from one extreme of its orbit to the other. Let P expreſs the total parallax of a fixed ſtar of the firſt magnitude, M the magnitude of the largeſt of the two ſtars, m the magnitude of the ſmalleſt*, and p the partial parallax to be obſerved by the change in the diſtance of a double ſtar; then will $p = \frac{m-M}{Mm}P$; and p being found by obſervation will give us $P = \frac{pMm}{m-M}$. An example or two will explain this ſufficiently. Suppoſe a ſtar of the firſt magnitude ſhould have a ſmall ſtar of the twelfth magnitude near it; then will the partial parallax

parallax in γ Draconis, if at moſt it had amounted to $2''$, it follows, that the angle OaE may nearly amount to 4 or $5''$ for any thing we can conclude to the contrary from thoſe obſervations.

* As M and m are here taken to expreſs the relative diſtances of the ſtars, in meaſures whereof the diſtance of the neareſt ſtar is taken as unity, thoſe who think the poſtulata on which theſe eſtimations are built cannot be granted, may ſtill uſe the following formulæ, if inſtead of the magnitudes M, m, they put their own eſtimations of the relative diſtances of the ſtars, according to any other method whatever they may think it moſt eligible to adopt; for the apparent magnitude of ſtars is here only propoſed as the moſt probable means we have of forming any conjectures about their relative diſtances.

we are to expect to see be $\frac{12 \times 1}{12 - 1}$ P; or $\frac{1}{12}$ths of the total parallax of a fixed star of the first magnitude; and if we should, by observation, find the partial parallax between two such stars to amount to $1''$, we shall have the total parallax P = $\frac{1 \times 1 \times 12}{12 - 1} = 1'',0909$. If the stars are of the third and twenty-fourth magnitude, the partial parallax will be $\frac{24-3}{3 \times 24} = \frac{21}{72}$ P; and if, by observation, p is found to be a tenth of a second, the whole parallax will come out $\frac{,1 \times 3 \times 24}{24-3} = 0'',3428$.

It will be necessary to examine some different situations. Suppose the stars, being still in the ecliptic, to appear in one line, when the earth is in any other part of its orbit between O and E; then will the parallax still be expressed by the same algebraic form, and one of the maxima will still lie at O, the other at E; but the whole effect will be divided into two parts, which will be in proportion to each other as radius − sine to radius + sine of the stars distance from the nearest conjunction or opposition.

When the stars are any where out of the ecliptic situated so as to appear in one line O*abc* at rectangles to OE, the maximum of parallax will still be expressed by $\frac{m - M}{Mm}$ P; but there will arise another additional parallax in the conjuction and opposition, which will be to that which is found 90° before or after the sun, as the sine (S) of the latitude of the stars seen at O is to radius (R); and the effect of this parallax will be divided into two parts; half of it lying on one side of the large star, the other half on the other side of it. This latter parallax, moreover, will be compounded with the former, so that

 the

the diſtance of the ſtars in the conjunction and oppoſition will then be repreſented by the diagonal of a parallelogram, whereof the two ſemi-parallaxes are the ſides; a general expreſſion for which will be $\sqrt{\overline{\frac{m-M}{2Mm}P}^2 \times \frac{SS}{RR} + 1}$: for the ſtars will apparently deſcribe two ellipſes in the heavens, whoſe tranſverſe axes will be to each other in the ratio of M to *m* (fig. 8.), and A*a*, B*b*, C*c*, D*d*, will be cotemporary ſituations. Now, if *b*Q be drawn parallel to AC, and the parallelogram *bq*BQ compleated, we ſhall have $bQ = \frac{1}{2}CA - \frac{1}{2}ca = \frac{1}{2}Cc = \frac{1}{2}p$, or ſemi-parallax 90° before or after the ſun, and B*b* may be reſolved into, or is compounded of, *b*Q and *bq*; but $bq = \frac{1}{2}BD - \frac{1}{2}bd =$ the ſemi-parallax in the conjunction or oppoſition. We alſo have R : S :: *b*Q : $bq = \frac{pS}{2R}$; therefore the diſtance B*b* (or D*d*) $= \sqrt{\overline{\frac{p}{2}}^2 + \overline{\frac{pS}{2R}}^2}$; and by ſubſtituting the value of *p* into this expreſſion we obtain $\sqrt{\overline{\frac{m-M}{2Mm}P}^2 \times \frac{SS}{RR} + 1}$, as above. When the ſtars are in the pole of the ecliptic, *bq* will become equal to *b*Q, and B*b* will be $.7071\,P\frac{m-M}{Mm}$.

Hitherto we have ſuppoſed the ſtars to be all in one line O*abc*; let them now be at ſome diſtance, ſuppoſe 5″ from each other, and let them firſt be both in the ecliptic. This caſe is reſolvable into the firſt; for imagine the ſtar *a*, fig. 9. to ſtand at *x*, and in that ſituation the ſtars *x*, *b*, *c*, will be in one line, and their parallax expreſſed by $\frac{m-M}{Mm}P$. But the angle *a*E*x* may be taken to be equal to *a*O*x*; and as the foregoing form gives us the angles *x*E*b*, *x*E*c*, we are to add *a*E*x*, or 5″ to *x*E*b*, and we ſhall have *a*E*b*. In general, let the diſtance

tance of the ſtars be d, and let the obſerved diſtance at E be D; then will $D = d + p$, and therefore the whole parallax of the annual orbit will be expreſſed by $\frac{DMm - dMm}{m - M} = P$.

Suppoſe the two ſtars now to differ only in latitude, one being in the ecliptic, the other, for inſtance, 5″ north, when ſeen at O. This caſe may alſo be reſolved by the former; for imagine the ſtars b, c, fig. 7. to be elevated at rectangles above the plane of the figure, ſo that aOb, or aOc, may make an angle of 5″ at O: then, inſtead of the lines $Oabc$, Ea, Eb, Ec, EP, imagine them all to be planes at rectangles to the figure; and it will appear, that the parallax of the ſtars in longitude muſt be the ſame as if the ſmall ſtar had been without latitude. And ſince the ſtars b, c, by the motion of the earth from O to E, will not change their latitude, we ſhall have the following conſtruction for finding the diſtance of the ſtars ab, ac, at E, and from thence the parallax P. Let the triangle $ab\beta$, fig. 10. repreſent the ſituation of the ſtars; ab is the ſubtenſe of 5″, that being the angle under which they are ſuppoſed to be ſeen at O. The quantity $b\beta$ by the former theorem is found $\frac{m - M}{Mm}P$, which is the partial parallax that would have been ſeen by the earth's moving from O to E, had both ſtars been in the ecliptic; but on account of the difference in latitude it will now be repreſented by $a\beta$, the hypothenuſe of the triangle $ab\beta$: therefore, in general, putting $ab = d$, and $a\beta = D$, we have $\frac{\sqrt{DD - dd} \times Mm}{m - M} = P$. Hence D being taken by obſervation and d, M, and m, given, we obtain the total parallax.

If the ſituation of the ſtars differs in longitude as well as latitude, we may reſolve this caſe by the following method. Let

Let the triangle $ab\beta$, fig. 11. reprefent the fituation of the ftars, $ab = d$ being their diftance feen at O, $a\beta = D$ their diftance feen at E. That the change $b\beta$ which is produced by the earth's motion will be truly expreffed by $\frac{m-M}{Mm}P$, may be proved as before, by fuppofing the ftar a to have been placed at α. Now let the angle of pofition $ba\alpha$ be taken by a micrometer*, or by any other method that may be thought fufficiently exact; then, by folving the triangle $ab\alpha$, we fhall have the longitudinal and latitudinal differences $a\alpha$ and $b\alpha$ of the two ftars. Put $a\alpha = x$, $b\alpha = y$, and it will be $x + b\beta = aq$, whence $D = \sqrt{\overline{x + \frac{m-M}{Mm}P}^2 + yy}$; and $\frac{\sqrt{D^2 - y^2} \times M^2m^2 - xMm}{m - M} = P$.

If neither of the ftars fhould be in the ecliptic, nor have the fame longitude or latitude, the laft theorem will ftill ferve to calculate the total parallax whofe maximum will lie in E. There will, moreover, arife another parallax, whofe maximum will be in the conjunction and oppofition, which will be divided, and lie on different fides of the large ftar; but as we know the whole parallax to be exceedingly fmall, it will not be neceffary to inveftigate every particular cafe of this kind; for, by reafon of the divifion of the parallax, which renders obfervations taken at any other time, except where it is greateft, very unfavourable, the forms would be of little ufe.

To finifh this theory, I fhall only add a general obfervation on the time and place where the maxima of parallax will happen.

* The pofition of a line paffing through the two ftars, with the parallel of declination of the largeft of them, may be had by the micrometer I invented for this purpofe in the year 1779, of whieh a defcription has been given in a former paper; whence, by fpherical trigonometry, we eafily deduce their pofition $ba\alpha$ fig. 11. with regard to the ecliptic.

When

Fig. 1.

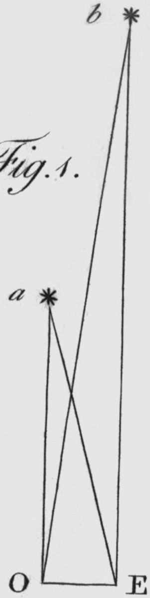


Fig. 2.

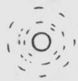


Fig. 3.

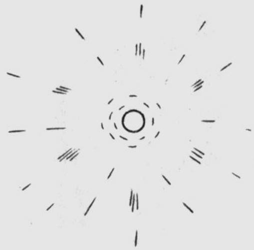


Fig. 4

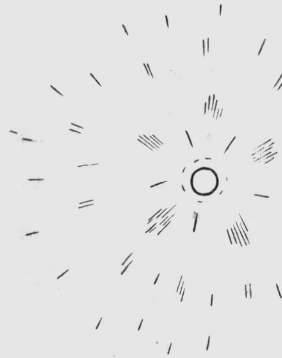


Fig. 5.

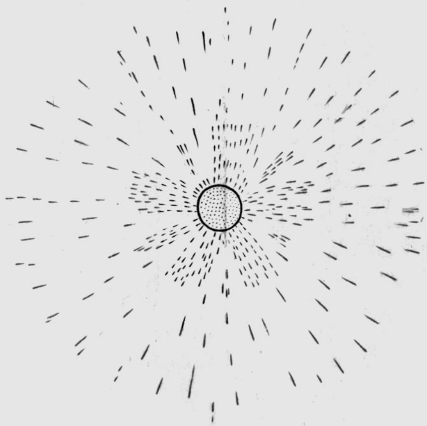


Fig. 6.

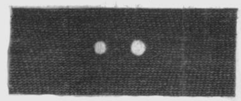


Fig. 4.

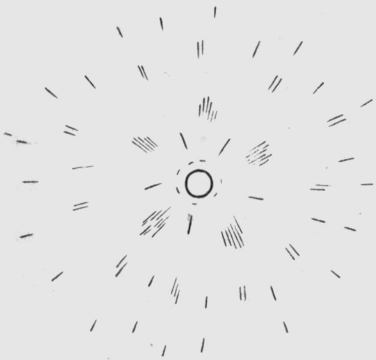


Fig. 6.

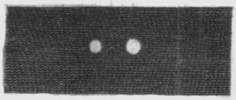
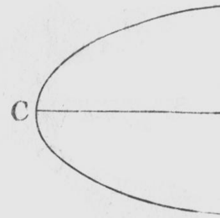


Fig. 7.



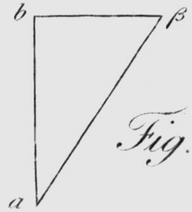


Fig. 10.

Fig. 12.

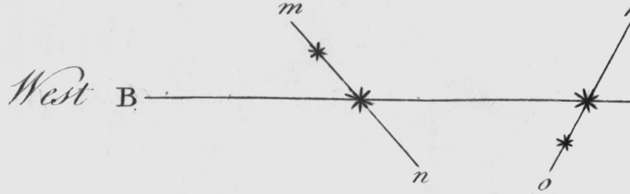


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Fig. 7.

Fig. 8.

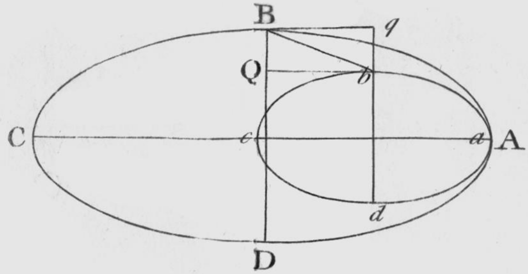


Fig. 9.

Fig. 10.

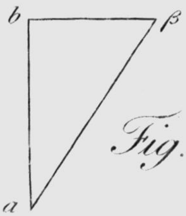


Fig. 11.

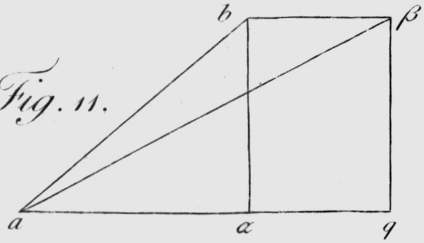
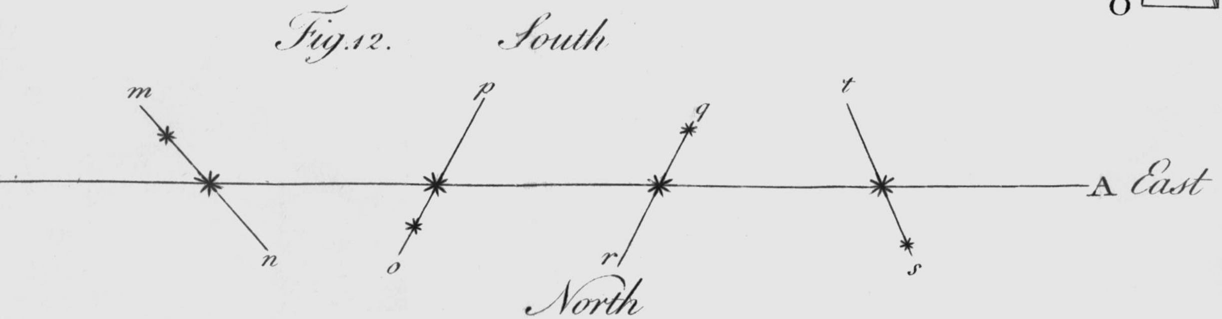


Fig. 12.



Basire S.

When two unequal ſtars are both in the ecliptic, or, not being in the ecliptic, have equal latitudes, north or ſouth, and the largeſt ſtar has moſt longitude, the maximum of the apparent diſtance will be when the ſun's longitude is 90° more than the ſtars, or when obſerved in the morning; and the minimum when the longitude of the ſun is 90° leſs than that of the ſtar, or when obſerved in the evening.

When the ſmall ſtar has moſt longitude, the maximum and minimum, as well as the time of obſervation, will be the reverſe of the former.

When the ſtars differ in latitudes, this makes no alteration in the place of the maximum or minimum, nor in the time of obſervation; that is to ſay, it is immaterial whether the largeſt ſtar has the leaſt or the moſt latitude of the two ſtars.